2. (8 marks)
   a) (5 marks) Give a pseudocode description of the algorithm implemented in your solution for Question #1.
   b) (3 marks) Derive the worst-case asymptotic, i.e., big-Oh, running time of your algorithm for Question #1.

2. (32 marks) Consider the following edge-weighted directed graph:

   a) (8 marks) Run Dijkstra’s algorithm (p. 595) on the directed graph above using vertex $v$ as the source vertex. In the style of Figure 24.6 in the textbook, show the $d$ and $\pi$ values and the vertices in set $S$ after each iteration of the while loop.

   b) (8 marks) Run the Bellman-Ford algorithm (p. 588) on the directed graph above using vertex $v$ as the source vertex. Relax edges in lexicographic order in each pass, and in the style of Figure 24.4 on the textbook, show the $d$ and $\pi$ values after each pass. Finally, please give the boolean value returned by the algorithm.

   c) (8 marks) Re-do part (a) with the weight of arc $(x,v)$ reset to -6.

   d) (8 marks) Re-do part (b) with the weight of arc $(x,v)$ reset to -6.
3. **(10 marks)** Consider the following problem:

**Dumbbell Subgraph (DS)**

*Input:* An undirected graph $G = (V, E)$ and two positive integers $k, l \geq 1$.

*Question:* Are there two cliques $C_1$ and $C_2$ and a simple path $P$ in $G$ such that $C_1$ and $C_2$ have $\geq k$ vertices apiece, $P$ has $\geq l$ edges, $P$ connects $C_1$ and $C_2$, the cliques and path do not have any edges in common, and the only vertices that $P$ shares with $C_1$ ($C_2$) is its connection-vertex?

Prove that this problem is $NP$-complete by (1) showing that this problem is in $NP$ and (2) giving a polynomial-time many-one reduction (algorithm + proof of correctness) to this problem from an $NP$-hard problem.