Science 1000: Lecture #5 (Wareham):

How We Move:
Robot Motion Planning

Moving in crowded space – is a puzzlement
Dealing with Intractability (Take II)

- What the $NP$-completeness contract really means:

  *if problem $A$ is $NP$-complete then*
  
  *There is no poly-time algorithm for $A$ that is deterministic and computes the best outputs for all inputs*

  *unless $P = NP$.*

- This contract only holds for algorithms that satisfy all of the listed conditions $\Rightarrow$ **practical algorithms that break one or more of these conditions are still possible, e.g., randomized, approximation!**

- Focus on what happens when we break the “poly-time” and “for all inputs” conditions.
Fixed-Parameter Tractability

Let’s relax our notion of tractability:

1. Focus on a set $P$ of one or more problem-aspects (parameters) whose values are small in practice.
2. Only consider inputs with small values for $P$.
3. Relax poly-time to fixed-parameter (fp-)time, i.e., run-time $f(P)n^c$ for some function $f$.

When the parameters in $P$ are small, fp-time is effectively poly-time, e.g., when $P = \{k\}$ and $k = 5$,

$$2^kn^2 \Rightarrow 2^5n^2 \Rightarrow 32n^2 \Rightarrow O(n^2)$$

Can prove fp-intractability with appropriate reductions and classes.
Fixed-Parameter Tractability (Cont’d)
The Cole’s Notes Version

<table>
<thead>
<tr>
<th></th>
<th>good</th>
<th>bad</th>
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<tbody>
<tr>
<td>classical complexity</td>
<td>poly-time solvable (Best)</td>
<td>$NP$-Complete</td>
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<tr>
<td>parameterized complexity</td>
<td>fp-tractable (Still OK)</td>
<td>fp-intractable</td>
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The Tractable Computation Thesis:
WHERE POSSIBLE, IMPORTANT PROBLEMS SHOULD BE SOLVED QUICKLY.

- Two conceptions of “quickly”:
  - quick in general (poly-time solvability)
  - quick under restrictions (fp-tractability relative to $P$)
- If a problem is intractable, look for restrictions to make it tractable.
- One way to do this is to look for parameters whose values are small in practice and then see if these restrictions yield fp-tractability.
Consider 3D motion planning in an obstacle-filled environment where we have to totally plan out a collisionless path from some initial robot-configuration $c_I$ to a final robot-configuration $c_F$, e.g.,
### 3D Robot Motion Planning

**Input**: An environment $E$ with obstacles, a robot $R$, and initial and final configurations $c_I$ and $c_F$ of $R$ in $E$.

**Output**: A sequence of moves of $R$ from $c_I$ to $c_F$ in $E$ that does not collide with an obstacle, if such a sequence exists, and special symbol $\perp$ otherwise.

- Is $PSPACE$-complete in general; however, robots often have a small number $k$ of joints (3 for robot arm, $\leq 20$ for robot hand).
- Unfortunately, is fp-intractable for parameter-set $\{k, X\}$, where $X$ is lots of other problem-aspects (Cesati and Wareham, 1995).
• Let’s step back to 2D motion planning and only require the robot to **react** from second to second based on what it sees to get from $c_I$ to $c_F$.

• Reactive “cockroach” robot:
2D Reactive Robot Adaptation

**Input:** An environment $E$ with obstacles, a reactive robot $R$, and initial and final configurations $c_I$ and $c_F$ of $R$ in $E$.

**Output:** A modified reactive robot $R'$ that can move from $c_I$ to $c_F$ in $E$ and does not collide with an obstacle, if such a robot exists, and special symbol ⊥ otherwise.

- Consider several types of allowable modification, e.g., change linkages between layers, add / delete layers relative to a library, add / delete layers in general.
Robot Motion Planning (Take II) (Cont’d)

• Is \( NP \)-complete in general, even for simplest types of modifications; however, reactive robots often have a small number \( l \leq 10 \) of layers.

• Unfortunately, is \( fp \)-intractable for parameter-set \( \{l, X\} \), where \( X \) is \textbf{lots} of other problem-aspects (Wareham et al, 2011).

• . . . However, is \( fp \)-tractable when the number of sensor-recognizable distinctions in the world is small, \textit{i.e.}, ignorance is (computational) bliss.

• Lots of work remains to be done . . .
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